INVITED SPEAKERS
Cubic graphs: some whereabouts of their perfect matchings, 2-factors and edge-colourings

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A graph with all vertices of degree 3 is said to be cubic. The study of cubic graphs is very vast mainly because important conjectures in graph theory such as the cycle double cover and Berge-Fulkerson conjectures will be true if proven for cubic graphs. Several kinds of problems that arise when studying perfect matchings, 2-factors and edge-colourings of graphs will be presented here. Partial solutions and infinite families with prescribed characteristics will be illustrated together with old and new results.

The goose of the golden differences is still alive

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Strong difference families (SDFs) have been implicitly considered extensively in the literature on combinatorial designs for more than one century but have been formally introduced for the first time only in 1999 [2]. After that, their explicit and systematic use [3, 5] apparently killed “the goose laying the golden eggs” (maybe better to say “golden differences”). As a matter of fact that goose seems to be in excellent health; indeed SDFs continue to produce a lot of results [1, 4, 6, 7, 8, 9]. In this talk I will try to explain the philosophy behind SDFs and I will select one of their most recent applications.

Keywords: (Strong) difference family; (resolvable) 2-design; Graph decomposition; Group action.

MSC: 05B05, 05B10

References

A few problems related to various combinatorial designs in terms of their maximality will be discussed.

One of the well examined cases concerns maximal partial latin squares. A *partial latin square* of order $n$ is an $n \times n$ array in which each cell is either empty or contains a single symbol from an $n$-element set $S$ such that each symbol occurs at most once in each row and at most once in each column. A partial latin square is *maximal* if no empty cell can be filled with an element of $S$ without violating latin conditions.

Evidently, a partial latin square of order $n$ corresponds to a proper-edge coloring of a balanced bipartite graph $H = (V, U, E)$ with at most $n = \chi'(K_{n,n})$ colors, where $|U| = |V| = n$. In this way considering maximal edge-colorings of non-bipartite graphs makes an obvious next step. Let $G$ be a graph of order $n$. A *maximal edge-coloring* of $G$ is a proper edge-coloring with $\chi'(K_n)$ colors such that no edge of the complement $\overline{G}$ can be attached to $G$ without violating...
conditions of proper edge-coloring. For given \( n \), a spectrum \( \text{MEC}(n) \) is defined to be the set of all sizes of graphs of order \( n \) which admit maximal edge-colorings.

Another class of objects to be considered includes partial Room squares. A partial Room square of order \( n \) and side \( n-1 \) on an \( n \)-element set \( S \) is an \((n-1) \times (n-1)\) array \( F \) satisfying the following properties:

1. every cell of \( F \) is either empty or contains an unordered pair of symbols from \( S \),
2. every symbol of \( S \) occurs at most once in each row and at most once in each column of \( F \),
3. every unordered pair of symbols of \( S \) occurs in at most one cell of \( F \).

A partial Room square is maximal if no further pair of elements can be placed into any unoccupied cell without violating the conditions that define a partial Room square. A spectrum \( \text{MPRS}(n) \) is the set of volumes of maximal partial Room squares of order \( n \), where the volume means the number of occupied cells.

In all of these cases, the common aim is to determine spectra.

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An overview of Heffter arrays

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In [1] Archeacon introduced the following notion of a Heffter array. Given a positive integer \( v = 2nk + 1 \), a Heffter array \( H(n;k) \) is an \( n \times n \) partially filled array with entries in \( \mathbb{Z}_v \) satisfying the following conditions: 1) each row and each column contains exactly \( k \) filled cells; 2) for every \( x \in \mathbb{Z}_v \setminus \{0\} \), either \( x \) or \(-x\) appears in the array; 3) the sum of the elements in every row and column is 0 (mod \( v \)). In this talk, besides presenting the most important existence results on this topic, see [2, 3, 6], I will focus on the applications of these arrays to difference families, orthogonal cycle decompositions, and biembeddings. Recent generalizations [5] and variants [4] will also be proposed.

References

3-uniform hypercycle systems

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A 3-uniform hypercycle of length \( k \) is a cyclic sequence of \( k \) vertices and \( k \) hyperedges, where the hyperedges are the consecutive triples of vertices. We study the problem of decomposing the edge set of the complete 3-uniform hypergraph into hypercycles of given length. We mostly discuss the case of 5-cycles.

From the history of Mixed Hypergraph Coloring

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We will discuss some results and open problems in Mixed Hypergraph Coloring. Related information can be found on Mixed Hypergraph Coloring website at http://spectrum.troy.edu/voloshin/mh.html
Contributed Talks
Graphs for Computer Science: an application in geo-spatial data analysis

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Due to the widespread diffusion of mobile and wearable devices which track movements of daily life, in the research field of Geo-spatial data analysis, appropriate Data Mining techniques are therefore necessary to extract significant information in a reasonable time from Big Data to process. In this talk, starting from a famous large-volume dataset that collects recorded GPS trajectories of different nature, three efficient strategies will be presented, based on the A-priori algorithm on grid mapping, in order to determine corridors common to multiple users. This algorithm examines the combinations of grid points that satisfy a minimum “traffic” threshold. The graphs will help us eliminate the discretization error due to the grid, filtering the outputs obtained in the previous step thanks to Radius Neighbor Graph, to obtain the final corridors. In computational geometry, the Fixed-Radius Near Neighbor problem is a variant of the Nearest Neighbor search problem.

The results of this challenging analysis could have many interesting applications, such as creating a social network to connect people to the same travel location, providing travel advice or reporting to the municipality the lack of public transport lines in some areas.

On Sequences with Distinct Partial Sums

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A subset $A$ of an abelian group $G$ is sequenceable if there is an ordering $(a_1, \ldots, a_k)$ of its elements such that the partial sums $(s_0, s_1, \ldots, s_k)$, given by $s_0 = 0$ and $s_i = \sum_{j=1}^{i} a_j$ for $1 \leq i \leq k$, are distinct, with the possible exception that we may have $s_k = s_0 = 0$. In the literature, there are several conjectures and questions concerning the sequenceability of subsets of abelian groups, which have been combined and summarized in [1] into the conjecture...
that if a subset of an abelian group does not contain 0 then it is sequenceable. The successful/partial resolution of this conjecture has implications in the study of graph decompositions and has applications to other combinatorial structures such as Heffter arrays.

In this talk, we will revisit the polynomial approach to sequenceability introduced in [4] to attack the problem on finite fields. In particular, we will show that Alon’s Combinatorial Nullstellensatz can be used to approach it also on cyclic groups whose order is not a prime and it is very effective for suitable weakenings of the problem.

References


Combinatorics meets choice theory: Bounded rationality is rare

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A choice function on a finite set $X$ of elements is a map which selects a single element from each nonempty subset of $X$. All models of bounded rationality developed in the last 20 years aim at explaining choice functions by means of properties of consistency, which are encoded by suitable second-order logic formulae. Most properties characterizing these models have a common feature, namely they are inherited by subchoices, but are not satisfied by at least one subchoice of most choices. We use the two notions of hereditary property and isomorphism of choices to prove two facts: (1) the fraction of choice functions
that can be explained by these models goes to zero as the number of elements tends to infinity; (2) even for small sets of alternatives, the fraction of choices that are boundedly rationalizable is trifling. The methods used in the proofs have a combinatorial flavor.

**Keywords:** Bounded rationality; hereditary property; choice isomorphism.

**References**


Configurations in Sicily before 1910 and after 1986

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My talk will discuss the research on configurations in Sicily before 1910 (historical part), followed by the investigation of configurations after 1986 which was started in the Acireale conference of 1986.

Configurations are linear regular uniform hypergraphs, mainly discussed in a geometrical language, and closely related to bipartite graphs, combinatorial designs and similar structures. A small but already quite interesting example is the Fano configuration with 7 points and 7 lines, corresponding to the Heawood graph. In order to exist the parameters of a configuration have to fulfill certain necessary conditions. In general, it has to be investigated whether these conditions are also sufficient, and if yes, how many non-isomorphic structures there are and what properties they have.

It was Vittorio Martinetti in Messina who in 1886 published his paper on configurations and started the “Sicilian research”. He later specialized on spatial configurations in the 1890s. Between around 1910 and 1986 there was a nearly perfect gap in original research on configurations.

The author started this research again in the Acireale conference in 1986 organised by Mario Gionfriddo. Not only in the further Sicilian conferences of 1989, 1992, 1998, and 2004, but throughout these years the author continued this research followed by a few other mathematicians. This development and the current state of knowledge will be considered, focusing on configurations and spatial configurations.

Steiner systems and configurations of points

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In this talk we introduce special configurations of reduced points in $P^n$ constructed from a Steiner System, combining Combinatorial Algebraic Geometry and Commutative Algebra. In particular, we associate two ideals, in a suitable polynomial ring, defining a Steiner configuration of points and its Complement.
We focus on the Complement of a Steiner configuration of points since it is a proper hyperplanes section of a monomial ideal that is the Stanley-Reisner ideal of a matroid. This connection allows us to study the homological invariants (Hilbert Function and Betti numbers) of the ideal $I_{X_C}$ of the Complement of a Steiner configuration. We also compute the parameters of linear codes associated to any Steiner configuration of points and its Complement.

This talk is dedicated to Lucia G. and Lorenzo. I also thank M. Gionfriddo, B. Harbourne, S. Milici and Zs. Tuza for their encouragement in finishing the project.

References


Characterizing subgeometries $PG(5, q)$ of $PG(5, q^2)$

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An algebraic variety $K$ can be thought as a set of points in a projective space that has a certain behavior with respect to subspaces. In a finite projective space $PG(d, q)$ the intersection varieties have a finite number of points. By combinatorial characterization of a variety we mean the classification of those sets of $PG(d, q)$ which, for axiom, possess a certain number of incidence properties of the given variety. The variety is intended to be characterized if one proves that a set satisfying the axiomatized properties is the variety, except at most some cases for particular values $q$ (sporadic cases). The characterization will be a solid finding when the required axioms are few, essential, significant and few also are the sporadic cases. In this talk we present a characterization of subgeometries $PG(5, q)$ of $PG(5, q^2)$. 
Regular two-graphs from strongly regular graphs

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In this talk we will give a classification of strongly regular graphs with parameters $(41,20,9,10)$ that have a nontrivial automorphism. We will talk about the construction of regular two-graphs with 42 vertices from these strongly regular graphs and about the construction of regular two-graphs with 38 vertices. We will present an enumeration of all regular two-graphs with 38 and 42 vertices that have at least one descendant whose full automorphism group is nontrivial.

From research to classroom: the case of $k$-permutations

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“Combinatorics might be considered the mathematical art of counting. Combinatorial reasoning is the skill of reasoning about the size of sets, the process of counting, or the combinatorial setting to answer the question ‘How many?’” (Hart and Sandefur 2018, p. VI). But, combinatorics is a field that most students find very difficult, since, for example, most combinatorial problems do not have readily available solution methods. Several studies over the years have promoted approaches to enhance capabilities in solving combinatorial problems, from primary children to middle- and high-school students. In this presentation, we will present an educational path that aims at addressing some difficulties with combinatorial problems by considering graph theory as a support in solving the problems. The activity has been inspired by Gionfriddo (2011). We will also present the results of the classroom activity that has been carried on in middle school. The whole analysis is conducted according to the five dimensions of the Teaching for Robust Understanding model (Schoenfeld 2014, 2016) and the Extended Modelling Cycle (Greefrath 2011).
A tour problem on a multidimensional toroidal array

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Tour problems on a chessboard have been studied in the past, due to the challenging problems that arise from this simple setting, the most famous of which is the knight’s tour problem, see [1,4].

In [2] a variation of this problem, called Crazy Knight’s Tour Problem, has been considered due to its applications to graph embeddings. In this case, the setting is a toroidal chessboard where some cells might be empty and cannot be visited. The goal is then to determine whether it is possible, under the action of a move function defined on every row and column, to visit exactly once every filled cell of the array.

In this talk, after having reported the main results obtained in [1], we will consider the more general instance of a multidimensional toroidal chessboard, and present some solutions to the Crazy Knight’s tour problem in this context.
References


Equitably 2-colourable cycle systems

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An \( \ell \)-cycle decomposition of a graph \( G \) is said to be equitably \( c \)-colourable if there is a \( c \)-vertex-colouring of \( G \) such that each colour is represented (approximately) an equal number of times on each cycle: more precisely, we ask that in each cycle \( C \) of the decomposition, each colour appears on \( \lfloor \ell/c \rfloor \) or \( \lceil \ell/c \rceil \) of the vertices of \( C \). In this talk, we consider the case \( c = 2 \) and present some new results on the existence of 2-colourable \( \ell \)-cycle systems.
Graph factorizations of the complete graph: results and open questions

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Let $H$ be a connected subgraph of a graph $G$. An $H$-factor of $G$ is a spanning subgraph of $G$ whose components are isomorphic to $H$. Given a set $\mathcal{H}$ of mutually non-isomorphic graphs, a uniform $\mathcal{H}$-factorization of $G$ is a partition of the edges of $G$ into $H$-factors for some $H \in \mathcal{H}$. In this talk we present results and open questions on the existence of a uniform $H$-factorization of $K_v$ or $K_v - I$.

Designs from parallel lines of an affine geometry

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I would like to present some new infinite series of $2-(q^n,k,q,\lambda)$ designs – obtained by difference methods – whose points are those of the affine geometry $AG(n,q)$ and whose blocks are union of $k$ parallel lines of this geometry.
Ghosts behind polynomials

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Abstract

The power sum polynomial of a (multi-)subset $S$ of $\text{PG}(n, q)$ is a homogeneous polynomial in $n+1$ variables, defined as the sum of the $(q-1)$-th powers of the Rédei factors associated to the points of $S$ [4]. Differently from the Rédei polynomial, a same power sum polynomial may be obtained from more than one set. Indeed, it turns out that two multi-subsets having the same power sum polynomial “differ”, in the multiset sum sense, by a multi-subset whose associated power sum polynomial is the null one. Such multisets are called ghosts, in analogy with the corresponding objects in discrete tomography [1].

In this talk we present some results about ghosts in $\text{PG}(2, q)$. We investigate the space of ghosts, compute its dimension and characterize some classes. Moreover, we explicitly enumerate ghosts for planes of small order.

The present talk is based on [2, 3].

References


On one property of the incidence graphs of 2-designs

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Let $\Gamma$ denote a bipartite graph with vertex set $X$ and color partitions $Y, Y'$, and assume that every vertex in $Y$ has eccentricity $D \geq 3$. For $z \in X$ and non-negative integer $i$, let $\Gamma_i(z)$ denote the set of vertices in $X$ which are at distance $i$ from $z$. Graph $\Gamma$ is 2-$Y$-homogeneous whenever for all $i$ ($1 \leq i \leq D-1$) and for all $x \in Y$, $y \in \Gamma_2(x)$ and $z \in \Gamma_i(x) \cap \Gamma_i(y)$, the number of common neighbours of $x$ and $y$ that are at distance $i-1$ from $z$ is independent of the choice of $x$, $y$ and $z$.

In this talk, we discuss the 2-$Y$-homogeneous condition of the incidence graphs of 2-designs. We prove that quasi-symmetric 2-designs that are quasi-symmetric 3-$(v, k, \lambda)$ designs with intersection numbers 0 and $y = \lambda+1$ are the only 2-designs which have 2-$Y$-homogeneous distance-biregular incidence graphs. Moreover, every 2-$Y$-homogeneous distance-biregular graph with eccentricity $D = 3$ is the incidence graph of such a design.

Cut-point sets of a finite simple graph

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Cut-point sets of a finite simple graph are special sets of vertices that disconnect the graph. They arise in Commutative Algebra in the study of binomial edge ideals, a class of ideals of a polynomial ring associated with finite simple graphs. Indeed, several algebraic properties and invariants of these ideals can be studied by looking at the cut-point sets of the underlying graph. In this talk I will introduce the notion of cut-point set and discuss a graph-theoretic problem that would imply a conjecture on binomial edge ideals.
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